Math 141
Joseph C Foster
Summer 2017
Midterm 1

Name: Solutions

May 18th, 2017
Time Limit: 55 minutes

This exam contains 7 pages (including this cover page) and 15 questions. The total number of points is 100. You have 55 minutes to complete the exam.

Read each question carefully. When specified, you must show **all** *necessary* work to receive full credit.

No calculator/phone/smartwatch allowed under any circumstances. Place these items in your bag, out of reach. Cheating of any kind will not be tolerated and will result in a grade of zero.

Question	Marks	Score	Question	Marks	Score
1	5		9	8	
2	5		10	8	
3	5		11	8	
4	5		12	8	
5	5		13	8	
6	5		14	8	
7	6		15	10	
8	6		Total	100	

1. (5 marks) True or False: $f(x) = x^2 + x + 5$ is even.

   A. True B. False

2. (5 marks) True or False: The derivative of $f(x) = \cos(x)$ is $f'(x) = \sin(x)$.

   A. True B. False

3. (5 marks) Fill in the blank: Let $f(x)$ be a function. Then $\lim_{x\to c} f(x) = L$ if for every $\varepsilon > 0$ there is a $\delta > 0$ such that

   $|f(x) - L| < \varepsilon$ whenever $0 < |x-c| < \delta$.

For questions 4-6, choose the best answer. There is only one correct answer but you may choose up to *two*. If you choose two and one of the answers is correct, you will receive 2 marks.

4. (5 marks) Choose one: $\displaystyle\lim_{x\to\infty} \frac{7x^2 + x + 4}{3x^2 + 8x} =$

   A. ∞

   B. 1

   C. $\frac{7}{3}$

   D. Does not exist.

5. (5 marks) Choose one: Let $f(x)$ and $g(x)$ be differentiable functions. The derivative of $y = f \cdot g$ is

   A. $\frac{dy}{dx} = \frac{df}{dx} \cdot \frac{dg}{dx}$

   B. $\frac{dy}{dx} = f \cdot \frac{dg}{dx} + g \cdot \frac{df}{dx}$

   C. $\frac{dy}{dx} = f + \frac{dg}{dx}$

   D. $\frac{dy}{dx} = \frac{df}{dx} \cdot \frac{dg}{dx} + f \cdot g$

6. (5 marks) Choose one: Let $f(x)$ and $g(x)$ be differentiable function. The derivative of $y = \frac{f}{g}$ is

   A. $\frac{dy}{dx} = \frac{g\frac{df}{dx} - f\frac{dg}{dx}}{g^2}$

   B. $\frac{dy}{dx} = \frac{g\frac{df}{dx} - f\frac{dg}{dx}}{\left[\frac{dg}{dx}\right]^2}$

   C. $\frac{dy}{dx} = \frac{f\frac{dg}{dx} - g\frac{df}{dx}}{f^2}$

   D. $\frac{dy}{dx} = \frac{f\frac{dg}{dx} - g\frac{df}{dx}}{g^2}$

In questions 7-17 you must show **all necessary work** to receive full credit. You do not need to simplify your final answer.

7. (6 marks) Find the domain and range of $f(x) = 1 - \sqrt{x}$.

Domain : $[0, \infty)$

Range : $(-\infty, 1]$

8. (6 marks) Use the Intermediate Value Theorem to explain why $f(x) = x^2 + \sqrt{2x+2} - 4$ has a root in the interval $[0, 2]$.

$f(0) = 0^2 + \sqrt{2(0)+2} - 4 = \sqrt{2} - 4 < 0$

$f(2) = 2^2 + \sqrt{2(2)+2} - 4 = \sqrt{6} > 0$.

Since f is continuous and $f(0) < 0$ and $f(2) > 0$, by the Intermediate Value Theorem, f contains a root in the interval $[0,2]$.

9. (8 marks) Show that the function $f(x) = \begin{cases} 3x+2 & x \le 5 \\ x^2 - 2x + 2 & 5 < x \end{cases}$ is continuous at $x = 5$.

$$\lim_{x \to 5^-} f(x) = \lim_{x \to 5^-} 3x+2 = 17$$

$$\lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} x^2 - 2x + 2 = 17.$$

Since $\lim_{x \to 5^-} f(x) = \lim_{x \to 5^+} f(x) = f(5)$, f is continuous at $x=5$.

10. (8 marks) Let $f(x) = 3x+1$. Use the $\varepsilon, \delta-$definition of a limit to show that $\lim_{x \to 2} f(x) = 7$.

Let $\varepsilon > 0$.

$|f(x) - L| < \varepsilon$ $\quad$ $0 < |x - c| < \delta$

$|3x+1-7| < \varepsilon$ $\quad$ $0 < |x-2| < \delta$

$|3x-6| < \varepsilon$

$3|x-2| < \varepsilon$

$|x-2| < \varepsilon/3$ $\quad$ Let $\delta = \varepsilon/3$

Thus $\lim_{x \to 2} f(x) = 7.$

11. (8 marks) Find the limit of $f(x) = \dfrac{\sqrt{x+1}-2}{x^2-9}$ as x approaches 3.

$$\lim_{x\to3} \frac{\sqrt{x+1}-2}{x^2-9} = \lim_{x\to3} \frac{x+1-4}{(x^2-9)(\sqrt{x+1}+2)} = \lim_{x\to3} \frac{x-3}{(x-3)(x+3)(\sqrt{x+1}+2)}$$

$$= \lim_{x\to3} \frac{1}{(x+3)(\sqrt{x+1}+2)} = \frac{1}{(3+3)(\sqrt{3+1}+2)}$$

$$= \underline{\underline{\frac{1}{24}}}$$

12. (8 marks) Let $f(x) = \dfrac{x\sin(x)}{2-2\cos(x)}$. Use the fact that $1-\dfrac{x^2}{6} \le f(x) \le 1$ to find $\lim\limits_{x\to0} f(x)$.

$$\lim_{x\to0} 1-\frac{x^2}{6} = 1$$

$$\lim_{x\to0} 1 = 1$$

Thus, by the Sandwich Theorem,

$$\lim_{x\to0} f(x) = \underline{\underline{1}}.$$

13. (8 marks) Use the limit definition of the derivative to calculate $f'(x)$ if $f(x) = 4 - x^2$.

$$\lim_{h \to 0} \frac{4-(x+h)^2-(4-x^2)}{h} = \lim_{h \to 0} \frac{-(x^2+2hx+h^2)+x^2}{h}$$

$$= \lim_{h \to 0} \frac{-2hx-h^2}{h} = \lim_{h \to 0} -2x-h = \underline{\underline{-2x.}}$$

14. (8 marks) Calculate the derivative of $f(x) = \dfrac{2x^2+5}{3x-2}$.

Quotient rule.

$$f'(x) = \frac{(3x-2)(4x)-(2x^2+5)(3)}{(3x-2)^2}$$

Simplified: $\dfrac{6x^2-8x-15}{(3x-2)^2}$

15. (10 marks) Calculate the derivative of $f(x) = \dfrac{(5x^2+13)(2x^3+3)^{11}}{7x-2}$.

$$g(x) = (5x^2+13)(2x^3+3)^{11}$$

$$g'(x) = (5x^2+13)\left(11(2x^3+3)^{10}6x^2\right) + (10x)(2x^3+3)^{11}$$ Product + Chain.

$$h(x) = 7x-2 \quad , \quad h'(x) = 7.$$

Quotient rule.

$$f'(x) = \frac{(7x-2)\left[(5x^2+13)(66x^2(2x^3+3)^{10} + 10x(2x^3+3)^{11}\right] - 7(5x^2+13)(2x^3+3)^{11}}{(7x-2)^2}$$

Simplified: $$\frac{(2x^3+3)^{10}(2380x^5 - 700x^4 + 5824x^3 - 1611x^2 - 60x - 27}{(7x-2)^2}$$

lol.